

# ATMOSPHERIC TRANSMISSIVITY: A MODEL COMPARISON FOR EQUATORIAL ANDEAN HIGHLANDS ZONE

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## A bstract

Ecuador is a South American country divided by the Equator. It is bordered by Colombia to the North, Peru to the East and South, and bounded on the West by the Pacific Ocean. Due to its location, Ecuador experiences very little variation in sunshine duration and very intense solar radiation. Moreover, about one third of the country is located at higher altitudes because of the presence of the Andes Mountain range, which means that those places receive higher solar radiation compared with lower regions of the country. However, there is not enough meteorological time series available for the entire region, especially for solar radiation. Nevertheless, in Riobamba (a city at 2750 m.a.s.l.), total solar radiation has been measured since 2007 with automatic meteorological stations and total sunshine hours, daily temperatures, precipitation, and wind speed since 1975 with a mechanical weather station. This data were used to calculate coefficients of atmospheric transmissivity by means of six well-known models: Prescott, Hargreaves, Garcia, Bristow–Campbell, Hunt, Richardson Reddy. The results show that the sunshine hour's model obtained the best performance among all the six models, indicating that this model could be used to estimate the incident solar radiation in the High Andean equatorial zone and to impute the missing data of global radiation in the zone. If data of sunshine hours is not available, the second best model uses temperatures and wind speed.

**Keywords:** Solar radiation, High Andean equatorial zone, model application.

## R esumen

Ecuador es un país ubicado en América del Sur dividido por la línea Ecuatorial. Al norte está limitado por Colombia, al sur y al este por Perú y al oeste por el Océano Pacífico. Debido a su localización, Ecuador percibe una pequeña variación en la duración de las horas de sol e intensa radiación solar. Además, alrededor de un tercio del país está localizado en las partes más altas debido a la presencia de la cordillera de los Andes lo cual significa que todos estos lugares reciben una alta radiación solar comparada con las regiones más bajas del país. Sin embargo, no existe una suficiente cantidad de series de datos meteorológicos disponibles de toda la región, especialmente de radiación solar. Favorablemente en Riobamba (una ciudad a 2750 m.s.n.m), los datos de radiación solar han sido medidos por una estación meteorológica automática desde el 2007 y los datos de horas de sol, temperaturas diarias, precipitación y velocidad de viento han sido medidos desde 1975 por una estación meteorológica manual. Estos datos han sido usados para calcular los coeficientes de transmisibilidad atmosférica de seis modelos conocidos: Prescott, Hargreaves, Garcia, Bristow–Campbell, Hunt, Richardson Reddy. Los resultados muestran que el modelo basado en las horas de sol ha obtenido el mejor rendimiento de los seis modelos, indicando que este modelo puede ser utilizado para estimar la radiación solar incidente en la Zona Ecuatorial Alto Andino y reemplazar los datos faltantes de la radiación solar en la zona. Si los datos de horas de sol no están disponibles, el siguiente mejor modelo está basado en temperaturas y velocidad de viento.

**Palabras claves:** Radiación solar, zona ecuatorial alta andina, aplicación de modelo.

## 1. INTRODUCTION

Using solar technologies could be very important for Ecuador since its geographical location and the presence of the Andes Mountains makes that the solar radiation is stronger than many other locations worldwide. In spite of their importance, solar radiation measurements in Ecuador are rare and infrequent, mainly due to the cost of specialized equipment and lack of trained personal.

In the existent meteorological stations, the main problem is that the data is frequently lost because the equipment is not frequently maintained and the personal in charge of taking the measurements is not properly trained. For this reason, it is important to perform statistical analysis to have a trustworthy database (1).

Many models has been developed to estimate solar radiation by using other (more common) meteorological variables such as sunshine duration hours, minimum and maximum air temperatures, precipitation and wind speed (2,3,4). Describes work global solar radiation covering a wide range of geographical and climate conditions; but there is not data over Ecuador.

The main objective of this work is to determine the most appropriate empirical coefficients that are used in the aforementioned models so that be possible to impute the missing solar radiation data in the databases.

## 2. METHODS

### 2.1 Data

Riobamba is a city situated in the central part of Ecuador at latitude  $1^{\circ} 40' 28''$  South, longitude  $78^{\circ} 38' 54''$  West and 2750 m.a.s.l. An automatic meteorological station was installed and operated in this city between June 2007 and De-

cember 2012; however, the pyrometer worked until April 2012. The station was registering and recording different parameters such as temperature, relative humidity, wind speed, atmospheric pressure and daily average total solar radiation. On the other hand, the National Meteorological Institute has installed a meteorological station in the same city and it has provided the daily total hours of sunshine, maximum daily temperature, minimum daily temperature, and other parameters for the same period of time.

The meteorological data was obtained using an automatic station located at latitude  $1^{\circ} 39' 17''$  South, longitude  $78^{\circ} 49' 39''$  West, and 2820 m.a.s.l. The available sensors are: pyrometer (Li-Co #LI-200SA) with certificate of calibration y and error of 5%., thermometer (NGR #110S), anemometer (NGR #40C) and pluviometer (Rain Gauge Tipping Bucket). The average values are stored every 10 minutes in a data logger NRG Symphonie. About 950 meters from this station, it is located a manual meteorological station at latitude  $1^{\circ} 39' 3''$  South, longitude  $78^{\circ} 41' 7''$  West, 2840 m.a.s.l. This station has a sunshine duration sensor Campbell-Stokes, which data is registered physically.

### 2.2. Detection of outliers

To detect the presence of mono-variant outliers in the data, it was performed a descriptive analysis using graphics and a robust standardization using the median, which is an estimator of the central position of the data and the MEDA, which is a robust estimator of the dispersion. This calculation was performed using eq.1.

$$\frac{|x_i - med(x)|}{MEDA(x)} > 4 \text{ or } 5$$

To detect the presence of multi-variant outliers, the minimum covariance determinant estimator was used. It helps to the detection of masked outliers (1). A similar solution was proposed by Peña and Prieto (5); in this method, the data is projected in specific directions so that they have high probability of showing outliers. On the other hand, the coefficient of kurtosis is an indicator of the presence of small groups of outliers; for this reason, the directions where the projected points have maximum and minimum kurtosis were calculated. Then, in these directions, all the values in all the directions of maximum and minimum kurtosis are tested with eq. 1. If the values surpass the value of 5, they are considered suspects of being outliers.

Once these suspects were identified, a vector of means

and a matrix of variances and co-variances of clean data were calculated in order to compute the distances of Mahalanobis of the suspected data, using eq.2. (1,6).

$$D_R^2(x_i - \bar{x}_R) = (x_i - \bar{x}_R) S_R^{-1} (x_i - \bar{x}_R)'$$

Once the distances of Mahalanobis were obtained for every suspect data, they were ordered from the minor value to the major value and they were tested to check if they can be incorporated to the main group of data or if they are separated, using a multi-variant inference that can be found in Peña and Prieto (5).

### 2.3. Extraterrestrial radiation models.

To calculate the extraterrestrial solar radiation, the angular movement of the Sun observed on the sky was analyzed. For this purpose, it is necessary to deduct the solar hour in function of the local hour; the solar noon is considered when the Sun passes over the observer's meridian. In addition, two corrections to the hour were performed: the first one consisted in subtracting four minutes for every degree of difference between the standard meridian of the country (-78.7°) and the meridian of place that is being analyzed (-75°). The second correction corresponds to the perturbations in Earth's rate of rotation (3,7).

The extraterrestrial solar radiation can be calculated with eq. 3 and eq. 4.

Where:

$\theta_z$  is the zenith angle, which determines the position of the sun with respect to a vertical line.

$$G_{on} = G_{sc} (1.000110 + 0.34221 \cos B + 0.001280 \sin B + 0.000719 \cos 2B + 0.000077 \sin 2B)$$

$$B = (n' + 1) \frac{360}{365}$$

Where:

$G_{on}$  is the extraterrestrial solar irradiation.

$G_{sc}$  is the solar constant.

$n'$  is the number of the day of the year (1 to 365).

### 2.4. Estimation of solar radiation models

In order to apply the models to estimate the incident solar extraterrestrial radiation on earth, it is necessary to calculate the theoretical incident extraterrestrial radiation ( $H_o$ ), using eq. 5, eq. 6 and eq. 7.

$$H_o = G_{on} (\cos \theta_z)$$

$$\cos \theta_z = \sin \phi \sin \delta + \cos \phi \cos \delta \cos \omega$$

Where:

$\theta_z$  is the zenith angle, which determines the position of the sun with respect to a vertical line.

$\phi$  is the latitude of the place. In this work -1.65°.

$\delta$  is the solar declination, which determines the angular position of the Sun at noon with respect to the horizontal plane.

$$\delta = \left( \frac{180}{\pi} \right) \left( 0.006918 - 0.399912 \cos(B) + 0.070257 \sin(B) - 0.006758 \cos(2B) + 0.000907 \sin(2B) - 0.002697 \cos(3B) + 0.00148 \sin(3B) \right)$$

#### 2.4.1. Hours of sunshine models.

In this work, the equation proposed by Prescott (eq. 8) was used. The variables are:  $H_o$  as the total calculated extraterrestrial irradiation,  $H$  as the incident irradiation,  $n$  as the effective sunshine hours,  $N$  as the theoretical sunshine hours, and  $a$  and  $b$  as the empirical coefficients (2, 8).

$$\frac{H}{H_o} = a + b \frac{n}{N}$$

It is important to mention that theoretical sunshine hours changes according to the place and the day of the year. In this work, it is calculated with eq. 9.

$$N = \frac{2}{15} \cos^{-1} (-\tan \phi \tan \delta)$$

#### 2.4.2. Maximum and minimum temperature models

To use the maximum and minimum temperatures, three models were applied: one proposed by Hargreaves (eq. 10) in 1982, another proposed by García (eq. 11) in 1994, and one proposed by Bristow Campbell (eq. 12). (2,9). All these models use  $\Delta T$  as the difference between the maximum and minimum temperatures. It is worth to mention that eq.11 includes sunshine hours

$$\frac{H}{H_o} = a + b \Delta T^{0.5}$$

$$\frac{H}{H_o} = a_b [1 - \exp(-b_s \Delta T^{c_s})]$$

$$\frac{H}{H_o} = a_s [1 - \exp(-b_s \Delta T^{c_s})]$$

#### 2.4.3 Models based on temperatures, precipitation, and wind speed.

To improve the estimation of the solar radiation based in temperatures, it has been established that models can be mo-

dified by adding other variables such as precipitation or wind speed. The model proposed by Hunt, based in extreme temperatures and precipitation (eq. 13), and the model proposed by Richardson – Reddy (10), that incorporates wind speed (eq. 14) have been used in this work.

$$\frac{H}{H_0} = a_H + b_H (\Delta T)^{0.5} + c_H T_{max} + d_H PR + e_H PR^2$$

Where:

$a_H, b_H, c_H, d_H, e_H$  are empirical coefficients obtained from a multiple lineal regression.

$T_{max}$  is the maximum temperature °C.

PR is the precipitation in mm.

$$\frac{H}{H_0} = a_R + b_R * T_{min} + c_R * T_{max} + d_R * PR + e_R * Ws$$

Where:

$a_R, b_R, c_R, d_R, e_R$  are empirical coefficients obtained from a multiple lineal regression, Ws is the wind speed in m/s.

#### 2.4.4 Models based on satellite images.

Another method that was tried was to use simple correlations between global solar radiation and variables measured with satellites. Two sources of data were analyzed. The first one is the GOES weather satellite which provides information of five spectral bands, including near infrared and water vapor (11). The spatial resolution of this data goes from one kilometer to four kilometers. On the other hand, the satellites Terra and Aqua are equipped with a Moderate-Resolution Imaging Spectroradiometer (MODIS) that captures data in 35 spectral bands (12). The spatial resolution is 250 meters. For its better spatial resolution, it was decided to use information from the MODIS sensor.

The variable selected for to calculate the simple correlations was the Normalized Difference Vegetation Index (NDVI), which is indicator used to estimate the quantity, quality and development of

vegetation, based on the reflection and emission of solar radiation on plants (13). Although the vegetation is affected by variables such as altitude, precipitation rate, among others, it has a strong relation with solar radiation.

Then, using monthly averages of solar radiation measured on field, and coupling them with monthly averages of NDVI, obtained from satellite images, a simple correlation was found to deduct an equation that allows estimating radiation in un-instrumented places.

## 3. RESULTS

### 3.1. Data

From the complete universe of data, for the calculation of the models the years 2009, 2010 and 2011 were used, which had to correspond to 1095 registers; however, because of damaged equipment, maintenance and maximum capacity of storing, the available data was 75% approximately. It is important to mention that for all the statistical analyses to be valid, the data must comply with the assumption that they have a multi-variant normal distribution. In this case, this assumption was valid for years 2009 and 2010. For 2011, the data has mono-variant normal distribution. This means that all the analyses performed to detect the outliers were valid. On the other hand, the validation of the models was performed with data taken from the years 2007, 2008 and 2012, which represents 33% of the whole registers.

### 3.2. Detection of outliers.

With the mono-variant and multi-variant techniques for the detection of outliers, the number of suspected data was 34. Later, calculating the distances of Mahalanobis, that has a Chi-squared distribution with 5 degrees of freedom, and using multi-variant inference, it was verified that the whole group of suspects are outliers; consequently, these registers were not used for the calculations.

### 3.3. Models for estimation of solar radiation.

Figures from fig.1 to fig. 3 show the plots of the universe of registers that were used for the calculation of the empirical coefficients.

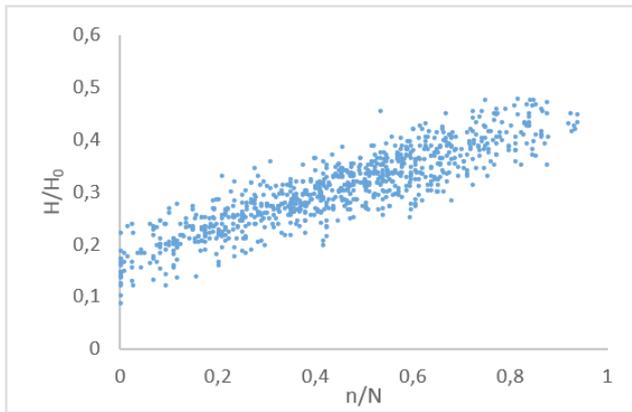


Figure 1: Atmospheric Transmittance vs. Ratio of sunshine hours (Model of Armstrong – Prescott).

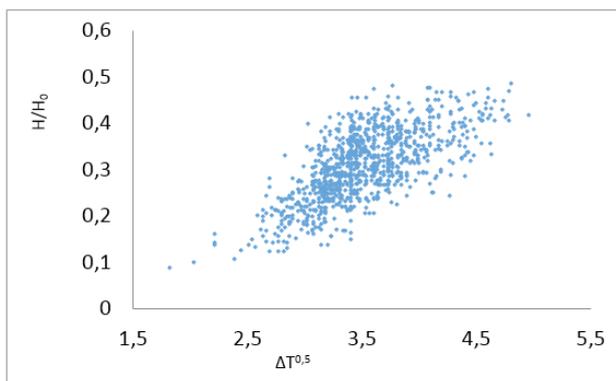


Figure 2: Atmospheric transmittance vs. square root of difference of temperatures (Model of Hargreaves).

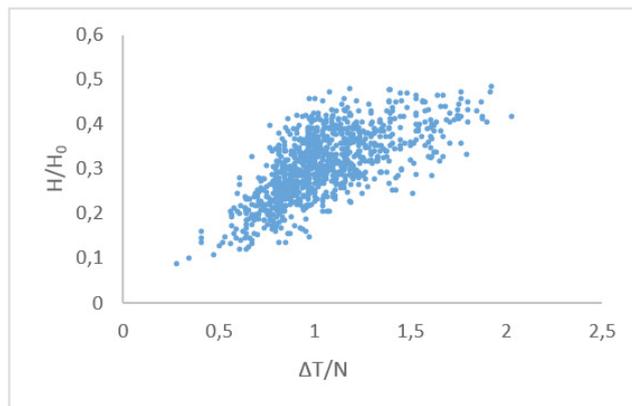


Figure 3: Model of García

A summary of the results of the empirical models is shown in Table 1.

Table 1. Empirical coefficients for the linear models.

Model	<i>a</i>	<i>b</i>	Standard Deviation	Error	Reliability	<i>r</i> <sup>2</sup>
Prescott	0.175	0.294	0.034	-0.013	90% - 95%	0.790
Hargreaves	-0.095	0.115	0.053	0.359	94% - 95%	0.474
García	0.112	0.188	0.054	0.202	89% - 95%	0.455

On the other hand, the other models are neither linear

nor mono-variable. The results of these models are presented in Table 2

Table 2. Empirical coefficients for the linear models.

Model	Dependency	Relation
Bristow – Campbell	Difference of temperature	$\frac{H}{H_0} = 0.469[1 - \exp(-0.317\Delta T^{0.5})]$
Hunt	Difference of temperature, Maximum temperature, Precipitation.	$\frac{H}{H_0} = 0.36 + 0.14\Delta T^{0.5} + 0.007T_{max} - 0.0027P$
Richardson Reddy	Minimum temperature, maximum temperature, precipitation, wind speed.	$\frac{H}{H_0} = -0.17 - 0.0007T_{min} + 0.027T_{max} + 0.049W$

To compare, for every model the coefficient of determination (*R*<sup>2</sup>) was calculated. These results are shown in Table 3

Table 3: Comparison of the results.

Coefficient of validation	Prescot	Hargreaves	García	Campbell	Hunt	Reddy
<i>R</i> <sup>2</sup>	0.790	0.474	0.455	0.509	0.555	0.662

The model that uses the sunshine hours as inputs has a better correlation with the trending line and smaller dispersion (standard deviation) and error than the models that use maximum and minimum temperatures, if a comparison between the linear models is made. In contrast, the model proposed by Reddy is the best one when the sunshine hours are not available.

For this reason, the empirical coefficients obtained from the Prescott model are recommended to estimate solar radiation from historical sunshine hours data for the Andean Highlands region in Ecuador.

Despite the fact that the model proposed by Prescott has the best correlation, information of sunshine hours is not widely available in Ecuador. For this reason, to test if the models can be used in other locations, the model proposed by Reddy was used in other weather stations to recover historical information. Figure 4 shows the locations of the weather stations. As the local conditions strongly influence the atmospheric transmissivity, individual coefficients had to be calculated for every location. The results are presented in table 4.

### 3.4. Models based on satellite images

As the measurement of solar radiation just began in January 2014, as an example of the procedure, one correlation was calculated using that month.

It is important to mention that not all the stations have to be used for the correlation. First, it is mandatory to eliminate the stations whose NDVI is false. For example, the station with the name “Espoch” cannot be taken into account because this station is located on concrete floor. This means that the NDVI of the exact location where the station is placed have a very low NDVI value, which is completely unrelated to the solar radiation of that place.

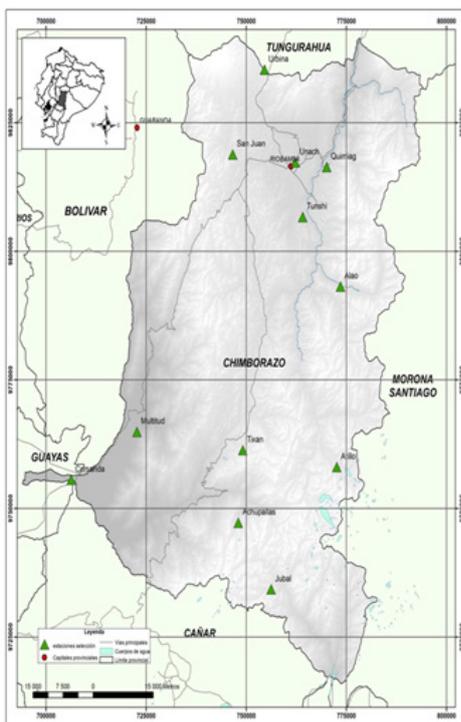
Table 5 shows the data used for the calculation of the correlation (includes “Espoch” station to demonstrated how unrelated are NDVI and solar radiation in that case) and Figure 6. shows the plot and correlation that was obtained.

**Table 4. Results of the model applied to other locations**

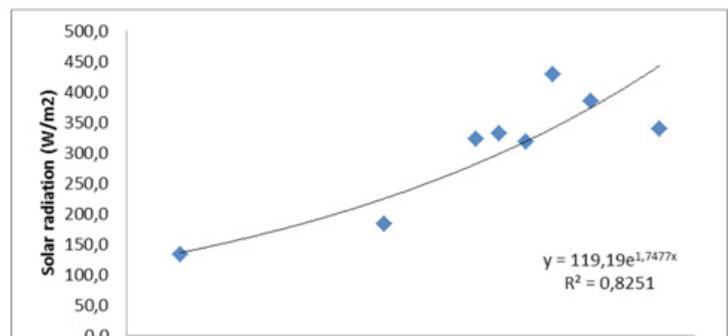
Models	Place	R <sup>2</sup>
$\frac{H}{H_0} = -0.18 + 0.02T_{max} + 0.07T_{min} + 0.05W_s$	Alao	0.62
$\frac{H}{H_0} = -0.30 + 0.03T_{max} + 0.01T_{min} + 0.05W_s$	Cumanda	0.73
$\frac{H}{H_0} = -0.32 + 0.03T_{max} + 0.02T_{min} + 0.22W_s$	Multitud	0.62
$\frac{H}{H_0} = -0.19 + 0.02T_{max} + 0.01T_{min} + 0.06W_s$	Quimiag	0.60
$\frac{H}{H_0} = -0.18 + 0.02T_{max} + 0.004T_{min} + 0.06W_s$	San Juan	0.59
$\frac{H}{H_0} = -0.17 + 0.01T_{max} + 0.02T_{min} + 0.02W_s$	Tixan	0.17
$\frac{H}{H_0} = -0.21 + 0.02T_{max} + 0.007T_{min} + 0.10W_s$	Tunshi	0.62
$\frac{H}{H_0} = -0.19 + 0.03T_{max} + 0.01T_{min} + 0.02W_s$	Urbina	0.58
$\frac{H}{H_0} = -0.28 + 0.04T_{max} + 0.01T_{min} + 0.04W_s$	Atillo	0.63
$\frac{H}{H_0} = -0.10 + 0.02T_{max} + 0.01T_{min} + 0.05W_s$	UNACH	0.68
$\frac{H}{H_0} = -0.11 + 0.02T_{max} + 0.005T_{min} + 0.05W_s$	Jubal	0.72
$\frac{H}{H_0} = -0.10 + 0.01T_{max} + 0.002T_{min} + 0.10W_s$	Achupallas	0.55

**Table 5. Data used for the calculation of correlation**

Nombre	NDVI	Average Solar radiation
Cumanda	0,36	183,9
Multitud	0,077	132,8
Quimiag	0,65	384,9
Espoch	0,29	406,9
Tunshi	0,60	427,6
Alao	0,53	331,3
Atillo	0,56	317,9
Tixan-Pistilli	0,49	323,1
Urbina	0,75	338,8



**Figure 4: Location of the weather stations where the models were tested.**



**Figure 6. Solar radiation vs. NDVI – January 2014**

## 4. DISCUSSION

According to Myers (14), Prescott recommends values of *a* between 0.17 and 0.43, and for *b* between 0.24 and 0.75. In addition, the values *a* = 0.29 and *b* = 0.42 were proposed by Frere (15) and cited by Baigorria (2) for the all Andean Highlands; these values are based in meteorological data of stations in the Andean Region of Peru. In this

work, for the model of Prescott, the obtained values are in the range of those proposed by Prescott. On the other hand, in comparison with the values proposed by Frere, the coefficients of this work are lower because the cloudiness in the Ecuadorian Andean region is more severe than the one found in the Andean region of Peru.

For the models based on the difference of temperatures, it is not possible to make a fair comparison because each region has particular meteorological conditions which affect the temperatures reached. Even if the cities have similar altitudes, the values presented by Baigorria for different cities in the Andean Region of Peru cannot be compared since the latitudes of the places are different.

Also, the coefficients calculated for the variable “precipitation” in the models of Hunt and Reddy are close to zero, which is an indication that this variable is irrelevant and it does not have influence in the estimation of the solar radiation. On the contrary, the incorporation of the variable “wind speed” in the model of Reddy improves the  $r^2$ , which is logic because the presence of wind a direct consequence of the interaction of solar radiation with atmosphere.

By looking at the results of using the model proposed by Reddy, it is clear that this model can be used safely used in other locations. The  $r^2$  shown in table 3 for this model is 0.662, and comparing his value with those shown in table 4, it can be seen that they are around the same value, except for “Tixan”, where the correlation is low.

On the other hand, the simple relation between monthly average solar radiation and NDVI has a high mathematical agreement. However, it is important not to forget that NDVI is influenced by many factors and that this method should be used only for preliminary estimations.

## 5. CONCLUSIONS

In summary, it had been demonstrated in this work that the best model to estimate the solar radiation based on other meteorological variables is the use of the sunshine hours. In addition, it is clear that a multivariate analysis is capital to detect, and separate the outliers to improve the estimation of the solar radiation with any model.

Moreover, it is clear that every region has particular meteorological conditions, which means that extrapolate the results of one region to estimate the radiation in other region may not be adequate, even if both regions share some characteristics such to be located in the Andean Region or to have similar altitudes.

Finally, the coefficient of correlation is high enough in some models to allow using their empirical coefficients to complete historical missing data of solar radiation in the Ecuadorian Andean region, so that researchers can use safely the values of solar radiation for their future work. In the case of using NDVI, every correlation should be calculated for every month, with their specific set of data to be able to extrapolate to other locations, and only if field information is not available to use other models.

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